COMPUTATION OF STEADY-STATE RADIAL DISTRIBUTION OF FINELY DISPERSED PARTICLES IN A VERTICAL TURBULENT GAS FLOW

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The technique and results of computation of the radial distribution of the concentration of solid particles suspended in an axisymmetric stabilized turbulent flow are discussed for different types of interaction of the particle flux with the walls of the channel.

The intensity of the transfer processes by gaseous suspension flows is to a large extent determined by the radial distribution of the particles in the flow. A subsonic axisymmetric flow of gaseous suspension with Stokes particles is investigated. The longitudinal distribution of the concentration has been investigated quite extensively [1]. The results of theoretical investigations of the transverse distribution of the particles are very incomplete, and in a number of cases they are contradictory (for example, see [2, 3]). This is apparently accounted for by the following factors: a) the use of any one mechanism of transfer of particles without sufficient justification (convective or diffusion) and also the investigation of only the core or the region of the flow near the walls; b) the absence of sufficiently reliable information for determining the diffusion coefficient of particles in the entire flow field (for example, see [4]); c) the lack or inadequate accuracy of computations of the field of the radial velocities of the particles as discussed in [5]; d) arbitrary choice of boundary conditions (for example, in [2, 6, 7]).

In the present work an attempt is made to evaluate the particle concentration field taking into consideration the above-mentioned peculiarities.

The concentration field is formed as a result of joint action of three particle fluxes: convective flux j_c, diffusion flux j_D, and the flux of particles at the flow boundary j_w:

$$\mathbf{j}_c \equiv \mathbf{u}_s \beta,\ \ \mathbf{j}_D = -D_s \operatorname{grad} \beta,\ \ j_w = c j^*. \tag{1}$$

In order to estimate the convective component of the radial mass flux it is necessary to know the local transverse time-averaged velocity of the particles v_s. It must be determined only from the system of equations of motion [5] with complete consideration of the basic force effects leading to the transverse displacement of the particles (thermophoresis, electrostatic charge, inertia, Saffman force, etc.). The numerical solution of this system, carried out by the authors, made it possible to obtain typical curves of the radial distribution of v_s [5].

The coefficient of diffusion of particles D_s can be found from the formula

$$\mathrm{Sc}^{-1} = \mathrm{Sc}_m^{-1} + \mathrm{Sc}_t^{-1}. \tag{2}$$

Here $\mathrm{Sc}_m^{-1} = 2kT/3\pi\rho\nu^2 d_s$ is the maximum value of the Schmidt number determining the Brownian diffusion; $\mathrm{Sc}_t^{-1} = \Psi(R, \tau_r)\nu^*/\nu$ is determined by the turbulent diffusion of particles. The turbulent viscosity coefficient ν^* is computed for the core of the flow and for the layer adjacent to the walls by the following formulas [8, 9]:

$$\nu^*/\nu = 6\cdot 10^{-4} y^3 \qquad (0 \leqslant y \leqslant 7.8); \tag{3}$$

$$\nu^*/\nu = 0.39\,(y - 7.07)(0.03 + 0.97R) \qquad (y > 7.8), \tag{4}$$

while the coefficient $\psi(R, \tau_r)$, which takes into consideration the inertia of the pulsating motion of the particles,

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is a function of the radial coordinate and the relaxation time of the particles [10]. The need for introducing a correction function Ψ is due to the passage of the particles from one vortex to the other, which leads to a certain decrease of D_{st} compared to ν^* in the core of the flow and possibly to an increase in the region adjacent to the wall. For this Ψ is different from 1. Quantitative estimates made for the cases $\Psi = 0.5$ and $\Psi = 2$ show that the particle concentration profile retains its character and changes only quantitatively, other conditions remaining unchanged. In view of this, and also due to the absence of direct data for carrying out quantitative estimates, in the first approximation Ψ is taken equal to 1 for the investigated fine particles.

The estimate of the particle flux j_w constitutes the main problem in different theoretical discussions [11, 12] (absolutely absorbing wall). According to [2], the determination of j_w according to $\beta_w = 0$ is not correct. The choice of $\beta_w = \text{const} \neq 0$ [7] is highly arbitrary, since in the general case this quantity must be determined from the solution of the concentration problem. It is better to use the coefficient c introduced in [13] as the probability of capture of the particles by the wall, giving an estimate of the ratio of j_w to the particle flux j^* arriving at the wall. In the more general case, the coefficient c must be taken as the ratio of j_w to the radial particle flux j^* having the same sign as j_w. If the wall is a source for particle mass, then $j_w < 0$ and j^* is the particle flux coming out of the wall. With this definition $0 \leq c \leq 1$, and specific values of this coefficient can be varied sufficiently arbitrarily, since they can be ensured by a suitable choice of the properties of the wall (adhesiveness, ideal elasticity, penetrability, roughness, etc.). In this sense, for a dispersed flow the coefficient c is essentially a quantity occurring in the boundary conditions. However, for the case of a completely penetrable (by particles) wall, the condition $c = 1$ is not sufficient and must be supplemented by information regarding one of the quantities j_w or β_w. It is obvious that the investigation of the processes occurring immediately at the wall (adhesion, cohesion, denudization, blowing, sucking, and so forth), necessary for determining c, is an independent complex problem which so far has not been adequately studied [14].

According to the law of conservation of mass of particles, in the stationary conditions we have

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R\left(\frac{1}{2}\operatorname{Re}BV_s - \operatorname{Sc}^{-1}\frac{\partial B}{\partial R}\right)\right] + \frac{\partial}{\partial X}\left(\frac{1}{2}\operatorname{Re}BU_s - \operatorname{Sc}^{-1}\frac{\partial B}{\partial X}\right) = 0, \tag{5}$$

where $B \equiv \beta/\beta_0$, $V_s \equiv v_s/u_0$, $R \equiv r/r_w$, and $X \equiv x/r_w$. Away from the entrance to the channel the concentration profiles may be assumed to be similar, i.e., $\partial B/\partial X \simeq d\bar{B}/dX = -2j_w/\beta_0\bar{u}_s$. Neglecting the diffusion transfer along the axis compared to the convective transfer we have

$$\frac{d}{dR}\left[R\left(\frac{1}{2}\operatorname{Re}BV_s - \operatorname{Sc}^{-1}\frac{dB}{dR}\right)\right] = 2R\frac{U_s}{\bar{U}_s}J_w, \quad J \equiv \frac{jr_w}{\nu\beta_0}. \tag{6}$$

For $B(r = 0) = 1$ the solution of this equation is of the form

$$B(R) = \exp\left[\int_0^R a(R)\,dR\right]\left\{1 - J_w\int_0^R b(R')\exp\left[\int_0^{R'} a(R)\,dR\right]dR'\right\}, \tag{7}$$

$$a(R) \equiv \frac{1}{2}\operatorname{Re}V_s\operatorname{Sc}, \quad b(R) \equiv \frac{\operatorname{Sc}\int_0^R u_s R\,dR}{R\int_0^1 u_s R\,dR}. \tag{8}$$

If $c = 0$ ($J_w = 0$), formula (7) permits direct computation of the concentration field. For $0 < c < 1$ the convective and diffusive fluxes have different directions. If $J^* = J_c$, then $J_w = cJ_c$; for $J^* = J_D$, from $J_c(1) + J_D(1) = J_w$ it follows that $J_D = J_c/(c - 1)$. Then in the general case we have

$$J_w = \frac{1}{2}kc\operatorname{Re}B(1)V_s(1), \quad k \equiv \begin{cases}1, & V_s(1) > 0,\\ (c-1)^{-1}, & V_s(1) < 0.\end{cases} \tag{9}$$

Using this expression for J_w in formula (7), written for $R = 1$, we determine the quantity $B(1) \equiv B_w$:

$$B_w = \left\{\exp\left[-\int_0^1 a(R)\,dR\right] + \frac{1}{2}kc\operatorname{Re}V_{sw}\int_0^1 b(R')\exp\left[-\int_0^{R'} a(R)\,dR\right]dR'\right\}^{-1}. \tag{10}$$

Expression (10) permits one to determine the particle concentration at the wall; then the quantity J_w and the concentration field can be computed from (9) and (7), respectively.

If the nature of the interaction of the particles with the wall is determined by the condition $c = 1$ (absolutely absorbing or penetrable wall), then $J_W = J^* = J_C + J_D$. In this case, when the quantity J_W is given directly, the computation of the field B(R) is done using expression (7). If in supplementing the condition c = 1 the value of B_W is given, then the solution of Eq. (6) has the form

$$B(R)=\frac{B_w\int_0^R b\exp\left(-\int_0^R a dR'\right)dR+\exp\left(\int_0^1 a dR\right)\int_R^1 b\exp\left(-\int_0^R a dR'\right)dR}{\exp\left(\int_R^1 a dR\right)\int_0^1 b\exp\left(-\int_0^R a dR'\right)dR}, \quad (11)$$

$$J_w=\left[1-B_w\exp\left(-\int_0^1 a dR\right)\right]\Big/\int_0^1 b\exp\left(-\int_0^R a dR'\right)dR. \quad (12)$$

Thus, formulas (7)-(12) with the corresponding specification of the boundary conditions permit one to compute the concentration field and the quantity J_W for all basic cases of interaction of particle flux with the wall. For example, since according to [15] the condition c = 1 is satisfied only for a specially prepared wall and under real conditions $0 \le c \le 1$ is more typical, in the subsequent analysis we shall consider only the case c < 1.

For the computation of the concentration field we use formulas (7)-(10). However, for determining the field Vs the system of equations of motion are solved beforehand under the assumption that B = 1 [5]. The obtained values of V_S(R) were used for numerical evaluation of the integrals occurring in (7), (8), and (10). The dependence B(R) thus determined was next used in the iteration refinement of the velocity field and the particle concentration field.

This procedure of computations was realized on an M-220 computer. The computations were carried out for the same conditions as in [5]: D = 0.1-1 m; u_0 = 1-10 m/sec; d_S = 0.1-1 μ; $\beta_0\rho_S/\rho = 10^{-2}$-$10^0$; q = 10^{-6}-10^{-4} C/kg. In the numerical analysis the air flow for P = 1 bar, T = 300°K, T_W/T_0 = 0.8-1.2, and $J_W \ge 0$ was taken into consideration. The specific difficulties in the computation of the concentration fields are related to the nature of the behavior of the diffusion coefficient D_S near the wall. As R → 1, D_S decreases sharply and the number Sc correspondingly increases. This leads to an increase of the absolute values of a and $\int_0^r a dR$ in the given zone. For certain variants of the initial data this leads to an overcrowding of the computer grid in the computation of the exponent of the integral, starting from a certain R* very close to unity.

If the value of the integral is large in absolute magnitude and negative, then in the region $R^* \le R \le 1$ we can put B = 0, since both terms in expression (10) are positive and increase sharply. If the value of the integral is positive and large, then for c = 0 formula (10) shows that the particle concentration at the wall tends to infinity. Physically, this means that a dense layer structure is produced at the wall, in which the limiting relative concentration is $B \approx 0.6\beta_0^{-1}$. For c ≠ 0 the first term in (10) is much smaller than the second, and in determining B_W this term can be neglected. The values of B(R) for R < R* are computed from (7) using the value of B_W already determined, while in the region $R^* \le R < 1$ the values of B(R) are computed by interpolation. Estimates show that for all investigated variants the errors caused by the above approximations are negligibly small. Furthermore, in the majority of cases R* is very close to unity and the interpolation intended for c ≠ 0 is necessary only for distances smaller than 5% of the thickness of the viscous sublayer.

The results of the computations enable us to elucidate the typical concentration fields for the investigated conditions (Fig. 1) formed under the influence of certain combinations of regime and boundary conditions. According to Fig. 1a the concentration field in the core of the flow is practically nongradient with a sharp increase of B in the zone adjacent to the wall. At the wall, the B_W may attain the limiting value, approaching the particle concentration in the dense layer. Such distributions of B are typical for conditions when there is no deposition of particles at the walls (c = 0, J_W = 0) and when the value of average transverse velocity of particles near the wall is positive and sufficiently large. For V_S the conditions are satisfied in those (investigated in [5]) cases when the force of the positive thermophoresis (curve 1), the electrostatic force and the positive thermophoresis (curve 2), and the effect of migration and Saffman force (3) are predominant. In all these cases the concentration distribution has the same nature as shown in Fig. 1a. This is accounted for by the fact that the resultant of all the forces acting on the particles and directed along the radius leads to a convective transport of particles to the wall. In view of J_W = 0, this flux must be compensated for by the diffusion transport from the wall. This nature of the concentration field agrees with experimental data, for example, [6].

For V_S < 0 and c = 0 the direction of the diffusion transfer changes, and in the presence of a nongradient

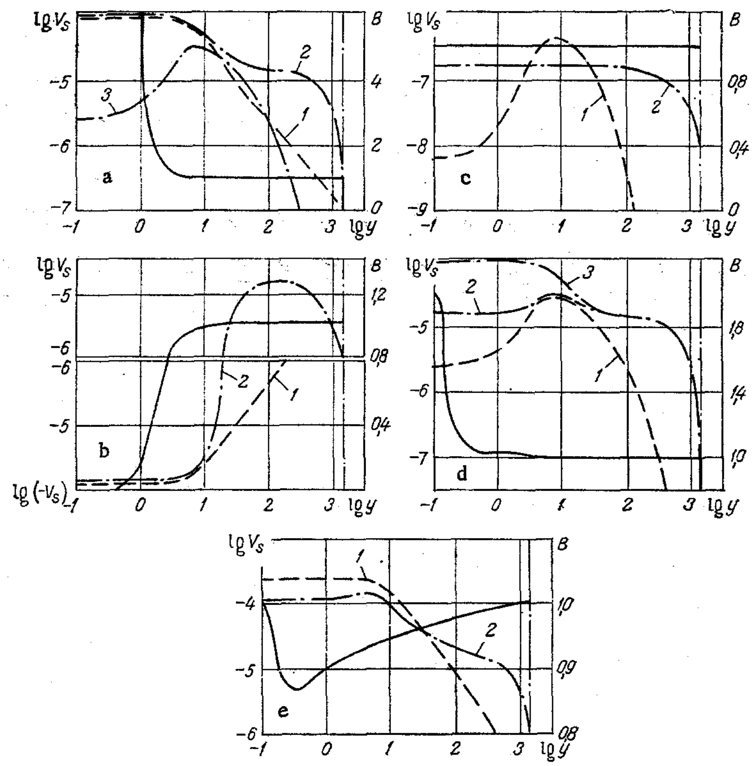

Fig. 1. Typical radial distributions of concentration $B = \beta/\beta_0$ (continuous lines) and the corresponding curves of the average transverse velocity of the particles $V_s = v_s/u_0$ (dashed curves 1), which are valid for the following values [5]: a–d) $Re = 6.2 \cdot 10^4$; $Sc_m = 3.3 \cdot 10^5$; $K_E = 3.25 \cdot 10^{-4}$; a,b,c) $Fr = 9.8$; $D/d_s = 10^6$; $Stk = 0.465 \cdot 10^{-5}$; d,e) $Fr = 9.8 \cdot 10^{-3}$; $D/d_s = 10^5$; $Stk = 0.465 \cdot 10^{-3}$; a) $T_w/T_0 = 0.9$, $c = 0$; b) respectively, 1.1 and 0; c) 1.0 and 0.99; d) 1.0 and 0.99; e) 0.8 and 0.99; dashed-dot lines (curves 2 and 3) show other types of curves of $V_s(R)$ for which the same radial distribution of concentration occurs.

nature of the field a sharp decrease of the concentration is observed in the core of the flow in the region near the wall (Fig. 1b). In the immediate neighborhood of the wall an almost complete absence of particles may be observed ($B_w \to 0$). The velocity of the particles V_s is formed due to the predominant effect of the negative force of thermophoresis (curve 1) or by the joint action of this force and the electrostatic force (curve 2) [5]. The results, for which the nonvariability of the concentration along the entire section is typical, are given in Fig. 1c. This nature of the curves is obtained for $0 < c < 1$, when the radial velocity of the particles is very small irrespective of the nature of the factors determining it. If the wall is absorbing ($c > 0$, $J_w > 0$) and the regime conditions of the gas suspension flow are similar to those in Fig. 1a, then the process of deposition of particles at the wall leads to a decrease of the concentration gradient near the wall (Fig. 1d).

For the conditions illustrated in Fig. 1e for c close to unity and for appreciable $V_s > 0$ in the region near the wall the abrupt change in V_s in transition from the core of the flow to the viscous sublayer (for example, due to the thermophoresis force) leads to the appearance of a minimum of $B(R)$ in this region. The common feature in Fig. 1a–e is that the concentration field in the core of the flow changes very slightly (in the range 5% of the values of β_0). This result is in good agreement with the data of [17, 18]. At the same time, in a thin region adjacent to the wall (excluding the case shown in Fig. 1c) a noticeable change in B occurs. This is explained, on the one hand, by the sharp decrease of the concentration of the coefficients of turbulent diffusion as the particles approach the wall and, on the other hand, by the significant increase of the contribution of the radial convective transport in the region near the wall. It should be noted that the intensity of the processes of transport of momentum, heat, and mass to the wall is to a large extent limited by the phenomena occurring near the wall. Therefore, in the computation of these processes it is necessary to consider the significant changes of the particle concentration in the zone of the flow adjacent to the wall, although it occupies only a small part of the cross section. Estimates of the thickness of this zone showed that the most significant variations of the field B(R) occur in a unique concentrated boundary layer whose thickness is smaller than the thickness of the viscous sublayer; a smoother variation of the concentration occurs also in the transition zone of the flow.

Only some results of the computation are presented in Fig. 1. However, these distributions of B(R) are quite typical for the motion of finely dispersed particles in an ascending turbulent gas flow. The formulas derived here for the computation of concentration fields in the segment of stabilized B profile can be used also for other conditions in which mechanisms determining V_s and Ψ differ significantly from those investigated here (for example, a significant effect of the inertia of the particles, their collisions, polydispersive nature, etc.). The study of the distribution of particle concentration in suspension flows in the initial segment of the flow is also of definite interest, but this is an independent problem.

NOTATION

c, coefficient similar to the probability of capture of particles by the wall; d_s, D, diameters of the particles and channel, m; D_s, diffusion coefficient of particles, m^2/sec; q, ratio of charge of the particle to its mass, C/kg; r, x, radial and longitudinal coordinates, m; T, temperature, °K; u, v, longitudinal and transverse velocities, m/sec; j, mass flux density normalized to particle density, m/sec; y, dimensionless distance from the wall, determined from the "wall law"; β, volume concentration of the particles in the flow; ε_0, dielectric constant in vacuum; ν, $\nu*$, molecular and molar viscosity coefficients, m^2/sec; τ_r, relaxation time of the particles, sec; ρ, density, kg/m^3; Reynolds number, $Re = u_0 D/\nu$; Froude number $Fr = gD/u_0^2$; Stokes number $Stk - Re \rho_s d_s^2 / 18 \rho D^2$; Schmidt number $Sc = v/D_s$; complex $K_E = qD^2 \rho_s \beta_0 / \varepsilon_0 u_0^2$. Indices: 0, the value at the axis of the flow; s, solid particles; t, turbulent analog; w, value at the wall of the channel; —, averaged over the cross section of the channel.

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